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14. ABSTRACT
The study concerns the dynamics of granular alignments held between rigid walls when the systems are subjected to continuous, large, forces at one end. It turns out that under such dynamical loading, granular chains exhibit peculiar quasi-periodic behavior. A deeper understanding of the physics associated with such behavior could lead to the development of multifaceted frequency sensors and energy harvesting technologies. The work also led to a deeper understanding of the contact forces between elastic grains with non-elliptic contact interfaces. In the course

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Report Title

Computational Study of Breathing-type Processes in Driven, Confined, Granular Alignments

ABSTRACT

The study concerns the dynamics of granular alignments held between rigid walls when the systems are subjected to continuous, large, forces at one end. It turns out that under such dynamical loading, granular chains exhibit peculiar quasi-periodic behavior. A deeper understanding of the physics associated with such behavior could lead to the development of multifaceted frequency sensors and energy harvesting technologies. The work also led to a deeper understanding of the contact forces between elastic grains with non-elliptic contact interfaces. In the course of the three years (2008-2011), this grant allowed us to support the work of two PhD students and one undergraduate who did an undergraduate honors thesis.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received	Paper
2012/01/24 1' 12	Robert P. Simion, Adam Sokolow, Surajit Sen. Nonlinear breathing processes in granular alignments, Applied Physics Letters, (12 2009): 0. doi: 10.1063/1.3268447
2012/01/24 1' 11	Alexander Breindel, Diankang Sun, Surajit Sen. Impulse absorption using small, hard panels of embedded cylinders with granular alignments, Applied Physics Letters, (08 2011): 0. doi: 10.1063/1.3624466
2012/01/24 1(9	Edgar Ávalos, Surajit Sen. How solitary waves collide in discrete granular alignments, Physical Review E, (04 2009): 0. doi: 10.1103/PhysRevE.79.046607
2012/01/24 1(8	Juan H. Agui, Surajit Sen, Robert L. Doney. Energy partitioning and impulse dispersion in the decorated, tapered, strongly nonlinear granular alignment: A system with many potential applications, Journal of Applied Physics, (09 2009): 0. doi: 10.1063/1.3190485
2012/01/24 1(7	S. Sen, R. P. Simion. Non-linear resonance-like processes in confined driven granular alignments and energy harvesting, Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, (06 2011): 0. doi: 10.1177/0959651811400940
2012/01/24 1(6	Chiara Daraio, Surajit Sen, Diankang Sun. Nonlinear repulsive force between two solids with axial symmetry, Physical Review E, (06 2011): 0. doi:
2012/01/24 1(4	Surajit Sen, T.R. Krishna Mohan. Dynamics of metastable breathers in nonlinear chains in acoustic vacuum, Physical Review E, (03 2009): 0. doi:
2012/01/24 1(3	Edgar A´ valos, Diankang Sun, Robert L. Doney, Surajit Sen. Sustained strong fluctuations in a nonlinear chain at acoustic vacuum: Beyond equilibrium, Physical Review E, (10 2011): 0. doi:
2012/01/24 1(1	T R Krishna Mohan, Surajit Sen. Linearity stabilizes discrete breaters, Pramana - Journal of Physics, (11 2011): 0. doi:

TOTAL: 9

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received	Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Invited Talks at Conferences

1. MRS Fall Meeting, Boston, Mass, November 2011 (unable to attend due to conflict)

2. Invited Speaker, ASME Conference (MECHMAT, Session on Granular Materials), Chicago, Illinois, June 2011 (denied).

3. Invited Speaker, 2nd Pan American/Iberian Meeting on Acoustics, Cancun, Mexico, November 15-17, 2010 (declined due to teaching commitments).

4. Invited Speaker, Particulate Materials in Extreme Environments (PMEE) 2010 Workshop, Lawrence Livermore National Labs, Livermore, CA, September 20-24, 2010 (canceled due to personal reasons).

5. Invited Speaker, SIAM Workshop on Nonlinear Dynamics, Philadelphia, PA, August 2010.

6. Invited Panelist, DARPA Workshop on Harvesting Energy, Blacksburg, VA, March 3-4, 2010.

7. Invited Speaker, Southern Workshop on Granular Materials, Vina del Mar, Chile, N0v 29-Dec 4, 2009.

8. Invited Speaker and Workshop Panelist, Triservice (Army, Air Force and Navy) Workshop on Mechanical Energy Harvesting, Virginia Polytechnic Institute and State University, Blacksburg, VA, August 12, 2009.

9. Invited Speaker, "Conference on Localized Excitations in Nonlinear Systems," Seville, Spain, July, 2009 (conflict with lecture at Powders and Grains 2009, my collaborator Dr T.R. Krishna Mohan from Centre for Mathematical Modeling and Computer Simulations, Bangalore, India delivered the invited lecture on my behalf).

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Invited Seminars and Colloqua

1. Seminar, Centre for Mathematical Modelling and Computer Simulations, Council of Scientific Investigation and Research, Government of India, Title: : "Newton's cradle, Fermi, Pasta, Ulam chain & the nonlinear many body frontier," June 29, 2011

2. Physics Seminar, Indian Institute of Science, Bangalore, India, Title: "Newton's cradle, Fermi, Pasta, Ulam chain & the nonlinear many body frontier," June 30, 2011

3. Physics Department Colloquium, SUNY Buffalo, Title: "Newton's cradle, Fermi, Pasta, Ulam chain & the nonlinear many body frontier," January 20, 2011.

4. Theoretical Physics Seminar, S.N. Bose National Centre, Title: "Modeling insurgency driven battles using Cellular Automata," February 5, 2009.

5. Physics Department Seminar, Presidency College, University of Calcutta, India, Title: "Nonlinear dynamics in granular materials," February 3, 2009

6. GALCIT Colloquium (Applied Physics and Aeronautical Engineering), California Institute of Technology, Pasadena, California, Title: Nonlinear dynamics in granular materials, October 31, 2008

==

Contributed Talks at Conferences

1. S. Sen, R.P. Simion and A. Sokolow
Granular Breathing, APS March Meeting, 2009, Pittsburgh, PA

2. R. Simion, S. Sen, A. Sokolow
Nonlinear breathing in compressed granular chains, APS March Meeting, 2010, Portland, OR

3. D. Sun, C. Daraio and S. Sen
Generalized Hertz law for grains with non-elliptic contacts
APS March Meeting, 2010, Portland, OR

4. Y. Takato and S. Sen
Stable solitary wwaves in granular alignments
APS March 2010 Meeting, Portland, OR

5. S. Sen and D. Sun
Collision between solitary waves in granular alignments
APS March 2011 Meeting, Dallas, TX

Number of Presentations: 20.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper
2012/01/24 1` 10	Eric C. Cyr, S. Scott Collis. CSRI Summer Proceedings 2010, CSRI Summer 2010. 2010/08/01 00:00:00, . : ,

TOTAL: **1**

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper
2012/01/24 1` 5	Diankang Sun, Edgar Avalos, Robert L. Doney, Robert P. Simion, Adam Sokolow, Surajit Sen. Nonlinear, Statistical and Applied Physics of Solitary Wave-like Objects in Granular Systems, Powders and Grains 2009. 2009/07/01 00:00:00, . : ,

TOTAL: **1**

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received	Paper

TOTAL:

Number of Manuscripts:

Books

Received	Paper

TOTAL:

Patents Submitted

1. Surajit Sen and Sourish Chakravarti, “Nonlinear granular mechanical device to dynamically extract dissipated energy,” ~~Provisional Patent applied for, September 2011 (Docket No. R-6667)~~

Patents Awarded

Awards

Election of Surajit Sen (PI) to Fellow of the American Physical Society
Citation -

For the discovery of how solitary waves break and secondary solitary waves form in granular media, for his leadership in organizing forums to represent and recognize the physicists from India and for raising consciousness about the problems and the importance of rural science education in India and the developing world.

December 2008

Named by Dean Maureen Grasso of the Graduate School, The University of Georgia, as a “UGA graduate degree holder who exemplify the intellectual legacy of our university.”

2009

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Diankang Sun	0.50	
Robert P. Simion	0.50	
FTE Equivalent:	**1.00**	
Total Number:	**2**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Surajit Sen	0.15	
FTE Equivalent:	**0.15**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED	Discipline
Yoichi Takato	0.00	
Aharon Festinger	0.00	
FTE Equivalent:	**0.00**	
Total Number:	**2**	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: ......

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:......

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:......

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): ......

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering: ......

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense ......

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: ......

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME

Diankang Sun

Robert P. Simion

Total Number: **2**

Names of other research staff

NAME	PERCENT_SUPPORTED

FTE Equivalent:

Total Number:

Sub Contractors (DD882)

Inventions (DD882)

5 **Nonlinear granular mechanical device to dynamically extract dissipated energy**

Patent Filed in US? (5d-1) Y

Patent Filed in Foreign Countries? (5d-2) N

Was the assignment forwarded to the contracting officer? (5e) N

Foreign Countries of application (5g-2):

5a: Sourish Chakravarty

5f-1a: State University of New York at Buffalo

5f-c: Department of Mechanical and Aerospace Engineering

Buffalo NY 14260

5a: Surajit Sen

5f-1a: State University of New York at Buffalo

5f-c: Physics Department, 239 Fronczak Hall

Buffalo NY 14260

Scientific Progress

We consider an unloaded axially aligned collection of elastic spheres confined within rigid walls, where   is allowed to vary between several and several hundred. Alignments with both monosized and progressively shrinking spheres (“tapered chains”) are considered. Being macroscopic, the spheres are assumed to incur restitutive losses when they interact and we adopt the well-established Walton-Braun scheme to model this restitution. These spheres are known to repel upon contact via an intrinsically non-linear repulsive force law upon contact due to Hertz (Hertz law). We show that when this system is driven at an edge sphere with some force and frequency, this driven dissipative system can respond via continuous sequences of overcompression followed by dilation. The frequency of this “breathing” process can be tuned across a wide range of values by varying the geometric and mechanical properties of the elastic objects, the tapering, and the system size and thus allows for resonance-like behavior of these nonlinear systems. We shall discuss how this system offers the potential to develop sensors that may be able to operate across a wide range of frequencies and could play a role in energy harvesting from a wide spectrum of sources such as ocean waves, wind and possibly geothermal sources.

Technology Transfer

THE FINAL REPORT TO THE US ARMY RESEARCH OFFICE ON

THE COMPUTATIONAL STUDY OF BREATHING TYPE PROCESSES IN GRANULAR MATERIALS*

SURAJIT SEN, PI,
Professor of Physics, Department of Physics, State University of New York at Buffalo, Buffalo, NY 14260-1500, USA
sen@buffalo.edu, ph: 716 907 4961, Fx: 716 645 2507

*In addition to the core work done as part of this proposed study on the breathing problem, several off-shoot problems were also addressed and those publications acknowledge the ARO support. These publications are provided separately.

January 2012

List of contents

Abstract

We consider an unloaded axially aligned collection of N elastic spheres confined within rigid walls, where N is allowed to vary between several and several hundred. Alignments with both monosized and progressively shrinking spheres ("tapered chains") are considered. Being macroscopic, the spheres are assumed to incur restitutive losses when they interact and we adopt the well-established Walton-Braun scheme to model this restitution. These spheres are known to repel upon contact via an intrinsically non-linear repulsive force law upon contact due to Hertz (Hertz law). We show that when this system is driven at an edge sphere with some force F and frequency Ω, this driven dissipative system can respond via continuous sequences of overcompression followed by dilation. The frequency of this "breathing" process can be *tuned across a wide range of values by varying the geometric and mechanical properties of the elastic objects, the tapering, and the system size and thus allows for resonance-like behavior of these nonlinear systems*. We shall discuss how this system offers the potential to develop sensors that may be able to operate across a wide range of frequencies and could play a role in energy harvesting from a wide spectrum of sources such as ocean waves, wind and possibly geothermal sources. The project has also led to two PhDs being granted, one to Dr Robert Paul Simion and the other to Dr Diankang Sun. Dr Sun's work was closely related to Dr Simion's focus but explored mathematical issues related to grain geometry. In addition, Aharon Festinger did an undergraduate honors thesis on trying to simulate how pulses penetrate into loaded 3D beds, an extension of Dr Simion's problem to 3D but at the pre-breathing level.

1. Introduction

Granular assemblies provide an example of media characterized by nonlinearity and discreteness [1-3]. Granular alignments, such as one of elastic spheres provide one of the simplest and richest dynamical systems that exhibit a wide range of unexpected dynamical behavior [1,4]. Hence these systems have been the object of intensive theoretical, simulational and experimental research since 1983 [1,4].

Impulse propagation through granular chains made of spheres varying in size, mass, stiffness continues to be a heavily investigated subject in the context of nonlinear dynamics [1,4]. Numerous studies have shown that alignments consisting of identical spherical grains, which initially barely touch one another, are capable of sustaining the propagation of non-dispersive lumps of energy, or solitary waves, when one of the edge grains is impacted by a striker [5-18]. If the mass of the striker has parameters (size, density) comparable to those of the impacted edge grain a single solitary wave will form in the chain. However, if the mass of the striker is much greater than the mass of the grain then multiple solitary waves with progressively decaying amplitude may eventually form in the chain out of an initial solitary wave train. Sokolow, Bittle and Sen [19] found numerically that in a granular alignment of unloaded, monodispersed elastic spheres of mass m and radius R a solitary wave train with P peaks is formed, when δ function perturbations are applied to P grains. This work has since been experimentally validated by Job *et al.* [20] More recent studies reveal remarkable shock absorption properties of small horizontal granular chains made of progressively decreasing spherical grains [21].

A granular chain in its simplest form consists of two elastic spheres of radii R_i and R_{i+1} subjected to an axial compression (without exceeding the elastic limits of the constituent materials). The repulsive Hertz potential [22], i.e., the potential energy stored at the contact between the two elastically overlapping spheres-can be written as:

$$V(\delta_{i,i+1}) = \frac{2}{5D_{i,i+1}}\sqrt{\frac{R_i R_{i+1}}{R_i + R_{i+1}}}\delta_{1i,i+1}^{5/2} , \tag{1}$$

where $D_{i,i+1}$ is a material dependent coefficient and $\delta_{i,i+1}$ represents the overlap between the two spheres:

$$\delta_{i,i+1} \equiv R_i + R_{i+1} - (x_{i+1} - x_i) \geq 0 . \tag{2}$$

When both the grains are made of the same material, the factor $D_{i,i+1}$ depends on the Poisson's ratio σ, and the bulk Young's modulus, Y, and $D = \frac{3}{2}(\frac{1-\sigma^2}{Y})$ and the derivative of the Hertz potential with respect to δ, the force felt at the interface between the two elastic beads, is

$$F(\delta_{i,i+1}) = \frac{1}{D}\sqrt{\frac{R_i R_{i+1}}{R_i + R_{i+1}}}\delta_{i,i+1}^{3/2} \equiv a_{i,i+1}\delta_{i,i+1}^{3/2}, \tag{3}$$

when the spheres are in contact with one another. If the spheres lose contact, the Hertz repulsion is no longer present [22] and the interaction force vanishes. For a weakly pre-compressed granular chain made of identical spheres in which a solitary wave has been initiated at one of its extremities via a short-lived perturbation - an impulse function, the speed of sound scales with $F^{1/6}$ [4].

While most of the studies, have been focused on understanding the impulse propagation through impacted granular chains via solitary waves, the driven granular systems, which have not yet been systematically probed, seem to present interesting nonlinear features leading to what we call nonlinear granular breathing [23]. Granular breathing and its possible consequences in the context of energy harvesting was investigated for chains constituted of identical (monodispersed) and progressively shrinking (tapered) spheres, when a *constant force* F is applied to one of the edge grains [23]. We assume that F is directed into the chain. For lossless monodispersed and tapered chains the product $TF^{1/6}$, which is approximately proportional to the length of the chain reads :

$$TF^{1/6} \propto R_{\text{large}}^{4/3}\left[\frac{1-(1-q)^{\alpha N}}{1-(1-q)^{\alpha}}\right]\rho^{1/2}D^{1/3}, \tag{4}$$

with $\alpha \approx 1/3$ (a numerically obtained result) where T is the breathing period – the time elapsed during one energy cycle, F is the magnitude of the applied force that acts on the first grain at all times and the tapering percentage q is defined as $q = 1 - R_{i+1}/R_i$. In earlier work, Sen *et al.* [24] have suggested that by coupling with appropriate sensors, these nonlinear features could be exploited in recovering/harnessing some of the energy released by the ocean waves as they come ashore.

In the present paper we analyze the dynamical behavior of driven granular alignments when an external force varying sinusoidally with time, $F = F_0 \sin\Omega t$, is applied to one of the edge grains (for monodispersed chains) or to the largest grain (for tapered chains). We present details about our simulations here on the monodispersed system and present the results of the tapered chain system without further elaboration on the tapered chain system itself.

We look for the "resonance-like" phenomena in these highly nonlinear systems in the presence of some dissipation. The reason why the sought out process differs from resonance itself is as follows. Unlike in normal resonance, the system's response in terms of how energy varies in time (recall this is a dissipative system) is not purely oscillatory but rather unique in nature, not describable by any simple known function that we are aware of. In addition, unlike in linear systems that are characterized by harmonic interactions between the particles, the system's response frequency depends on the chain geometry, the material medium and how the amplitude of the external driving force could influence the behavior of the system.

The restitutive losses are described via the quantity w as follows according to the Walton-Braun scheme [25]

$$\frac{F_{unloading}}{F_{loading}} = 1 - w, \tag{5}$$

where $F_{unloading} \leq F_{loading}$ and where $F_{unloading}$ denotes the force between two adjacent beads as they depart one from another and $F_{loading}$ is the force between the two grains as they press one against another [25]. When the equality holds true there is no dissipation in the system while if $F_{unloading} < F_{loading}$ the energy is primarily lost during the loading phase. For a certain frequency of excitation at or near one of the natural frequencies of the system damping is of primary importance,

2. Newtonian dynamics simulations

Our systems are modeled as horizontal chains of identical or progressively shrinking spheres with their centres axially aligned, made of the same material type and placed between two rigid flat boundaries. At the instant $t = 0$, when the spheres barely touch one another (no Hertz repulsive interaction), a time-dependent acceleration is imparted to one of the edge grains. The nearest neighbor interaction of the $N-2$ identical spherical grains (without the edge grains) is described by the following coupled nonlinear equations:

$$m_i \ddot{x}_i = \frac{5}{2}(a_{i-1,i}\delta_{i-1,i}^{3/2} - a_{i,i+1}\delta_{i,i+1}^{3/2}), \qquad i = 2, \ldots, N-1, \tag{6}$$

where m_i is the mass of the sphere, x_i denotes the position of the center of mass of grain i, and a_i is an adjustable parameter which depends on the geometrical and material properties of the grains. As initial conditions we consider:

$$x_i(t=0)=0, \quad \ddot{x}_i(t=0)=0, \qquad\qquad i=1,\ldots,N$$

$$\ddot{x}_1(t=0)=\frac{F_0 \sin\ (\Omega\ \ t)}{m_1}, \qquad \ddot{x}_j(t=0)=0, \qquad j=2,\ldots,N \tag{7}$$

$$R_{i+1}=R_i(1-q) \qquad\qquad i=1,\ldots N$$

where $t\in[0,t_{\max}]$ denotes the time variable, $t_{\max}$ sets the maximum length of our runs, F_0 is the magnitude of the applied force , Ω represents the driving frequency, m the mass of the sphere and the dots over a variable means its time derivatives. R_1 is the radius of the largest sphere .

The equations of motion are numerically solved via the velocity Verlet algorithm [26]. The position, velocity and acceleration of each grain in the system are recorded thus the kinetic, potential and total energy of the system are obtained. We have used different time steps of integration and settled for $\Delta t=10^{-5}$ μs, as to make a compromise between accuracy and efficiency and our runs extended over 10^4 μs. Different external sinusoidal forces were applied to different chains with different geometries. Metallic grains with low dissipation coefficients have been the object of numerous experimental studies so we chose Ti_6Al_4V ($\rho=4.42$ mg/mm^3, $\sigma=0.34$ and E = 110GPa) as the material type simulated in this study [27]. We have also simulated and obtained similar results for stainless steel and SiC. In analyzing small systems, $15\le N\le 30$ reasonable values have been assumed for the restitution coefficient ($0<w<<1$) and the radii of the grains (5-10mm) such that the results of this study could be compared to the results obtained from a real experiment.

3. Breathing in driven chains (same material medium)

First we analyzed the dynamical behavior of a monodispersed chain ($R_i = R, m_i = m$), with one of the outer grains subjected to an external time dependent force, $F = F_0 \sin \Omega t$.

For a specific chain configuration driven with a force of magnitude F_0 we tried various driving frequencies Ω to set the preliminary stages in finding the nonlinear resonance design parameters (number of grains, particles radii, mass densities, and material type). Phase diagrams which are useful tools in investigating and understanding the dynamics of nonlinear systems were the starting point of our work. In a Hertzian system (described by Eq. (1)) a significant part of the input energy is stored as potential energy at the grain-grain and grain-wall contact surfaces. Figure 1 shows the space-time kinetic energies plots for a system driven at different forcing frequencies. The chain comprises a small number ($N = 15$) of identical elastic grains and dissipative effects account for the energy lost. In each plot the horizontal axis indicates the particle site, the vertical axis shows the time elapsed and the shading gives the kinetic energy of each particle. The darker the spot the more kinetic energy the corresponding particle has. When the energy is supplied to the system at the same rate as the one as at it is being lost the resonance phenomenon occurs.

One can recognize that not all of the plots in Figure 1 depict an ordered pattern of the particles' energies - a behavior expected to be found at resonance. The only energy density plot which shows a repetitive pattern closer to the expected resonant behavior is the central plot in Figure 1. It can clearly be seen that in the central plot the particles are moving in a certain fashion as opposed to the other plots in which the particles' motion is somehow hard to trace.

Once the conditions for defining the resonance-like behavior in our nonlinear systems were set we plotted in Figure 2 the temporal behavior of the system. The total kinetic and potential energies resonate at different frequencies, which are results of the fact that a granular chain is not a conservative system, energy is continually exchanged with the driving mechanism, and energy is being transferred to the medium and lost via restitutional losses. The total energy of the system rises, reaches a maximum and then falls in a repetitive pattern (the total energy peaks have almost the same magnitude). When dealing with macroscopic discrete systems we do not expect to have only an exact value of the resonance frequency. However, if the driving frequency does not closely match the required resonance frequency (is either well above or below) the system does not experience a resonant behavior, as shown in Figure 2.

The next question we asked was how the driving frequency relates to N, the number of spheres making up the chain, when all the other parameters (F_0, R, q, w, ρ, D) are kept constant. Our numerical results show that when only the number of the grains changes the *driving* frequency changes according to the following relation:

$$\Omega_1 N_1 = \Omega_2 N_2 \text{ and } \Omega \propto \frac{1}{N} \tag{7}$$

We have also investigated the relation between the driving frequency Ω and F_0, the magnitude of the driving force for a constant configuration (N, R, q, w, ρ, D). Our simulational results show (Figure 3) that Ω changes with F_0 according to the following relation:

$$\frac{F_{01}^{1/6}}{\Omega_1} = \frac{F_{02}^{1/6}}{\Omega_2} \text{ and } \frac{F_0^{1/6}}{\Omega} = const \ , \ \Omega \propto F_0^{1/6} \ . \tag{8}$$

We have found that when only F_0 and N vary, the following relation holds true:

$$\frac{F_{01}^{1/6}}{\Omega_1 N_1} = \frac{F_{02}^{1/6}}{\Omega_2 N_2} \text{ and } \Omega \propto \frac{F_0^{1/6}}{N} \ . \tag{9}$$

The energy transport through a chain of equal sized spheres depends highly on the radii of the grains. If only the radius is the adjustable parameter then:

$$\frac{\Omega_1}{\Omega_2} = \frac{R_2^{4/3}}{R_1^{4/3}} \text{ and } \Omega \propto \frac{1}{R^{4/3}}. \tag{10}$$

By using all the above results, we find that for a granular alignment of identical spheres (same size and same material type) subjected to a time-dependent force, the driving resonance frequency reads:

$$\Omega \propto \frac{F_0^{1/6}}{NR^{4/3}}. \tag{11}$$

One would certainly expect that Ω depends on the characteristic properties of the material medium (D,ρ) to transfer the energy. The energy transport depends on both on the inertial property of the medium (to store kinetic energy) and the elastic property of the medium (to store potential energy). If the spheres are made of the same material type, we can guess that the inertial property is the mass density ρ. For the elastic property we can assume that the pre-factor D, which contains the Young's modulus and the Poisson's ratio, would be a suitable candidate. Mathematically this could be expressed in he following form:

$$\frac{F_0^{1/6}}{\Omega NR^{4/3}} = f(D,\rho,w), \tag{12}$$

where the function $f(D,\rho,w)$ depends on the material medium. Using dimensional analysis we conclude that:

$$\frac{F_0^{1/6}}{\Omega NR^{4/3}} = const(D,\rho,w) = f(w,\rho)D^{1/3},$$

and continuing our dimensional analysis we arrive to the following relation:

$$\frac{F_0^{1/6}}{\Omega N R^{4/3}} = k(w) D^{1/3} \rho^{1/2} \quad . \tag{13}$$

where $k(w)$ is a dimensionless coefficient that depends only on the restitution coefficient w.

4. Breathing in driven chains (different material medium)

For two different simulated materials (Ti_6Al_4V and stainless steel) with the same force amplitude (F_0), and the same N having the same radii (R) the following relation applies:

$$\left(\frac{\rho^{-1/2}}{\Omega} D^{-1/3} \right)_{Ti_6Al_4V} = \left(\frac{\rho^{-1/2}}{\Omega} D^{-1/3} \right)_{steel} . \tag{14}$$

We have carried out our extensive dynamical studies in the presence of restitutive effects with the restitution coefficient ($w = 0.01$) defined as in Eq. (5). From our simulations it could be inferred that when two monodisperse granular chains are made of different materials:

$$\left(\frac{F_0^{1/6} \rho^{-1/2}}{\Omega N R^{4/3}} D^{-1/3} \right)_{mat\,1} = \left(\frac{F_0^{1/6} \rho^{-1/2}}{\Omega N R^{4/3}} D^{-1/3} \right)_{mat\,2} = k(w) \ , \tag{15}$$

where $k(w)$ is a constant for a given w .We found $k \approx 3.685$ for $w = 0.01$. To clarify how a change in w could influence the resonance behavior more studies are under way. At present our investigations do not reveal any problems regarding the validity of Eq. (15) for values of w slightly greater than 0.01.

For a monodispersed chain obeying to the conditions stated above our studies indicate that the driving (resonance) frequency is of the form:

$$\Omega = \frac{F_0^{1/6}}{N R^{4/3} \rho^{1/2} D^{1/3}} \frac{1}{k(w)} \ . \tag{16}$$

We mentioned before that if the frequency of excitation is either above or below the resonance frequency, we expect that the system exhibit a somewhat chaotic response pattern. For a linear system, when the external force has a certain forcing frequency of oscillation, the steady-state response will have the same frequency of oscillation. However a nonlinear system could presumably exhibit chaotic behavior at frequencies below and above the resonance-like frequency.

If the magnitude of the external force and the topology (size, material type) of the system are not altered, we find that more than one driving frequencies can induce the expected resonance behavior in a granular chain.

The panels in Figures 4 and 5 show that only at or near certain values of the driving frequency the external source supplies the system with energy at a rate almost equal to that lost through dissipative effects. The shape of the total energy peaks change with Ω in an unexpected manner. Note that these results are not numerical artifacts but are due to the discreteness and nonlinearity of these systems and we are not in a position now to analytically describe the nonlinear breathing processes described in this work.

We find these unexpected results only for or near certain driving frequencies, and as we lower the frequency the number of sub-peaks in each peak in the energy versus time plot increases. We tried to find a relation between the number of peaks η and the resonance frequency Ω and in order to simplify the analysis we assumed the simple form:

$$\Omega = \frac{\beta}{R_{large}^{4/3}} \frac{1}{k(\eta, w)} \frac{1}{f(q, N)} , \tag{17}$$

where $\beta \equiv F_0^{1/6} / \rho^{1/2} D^{1/3}$ is a constant, $k(\eta, w)$ is a function that depends only on the number of the peaks for a given value of $w = 0.01$.The function $f(q, N) = \frac{1-(1-q)^{N/3}}{1-(1-q)^{1/3}}$, which from our previous results is a constant for a given configuration (N, q) turns to N in the limit $q \to 0$ (monodispersed chain) and we recover Eq. (16) with a slightly different value of k.

Our approach was to first calculate the approximate value of Ω for a monodispersed chain by using Eq (16), then to tune / adjust the computed value to the resonance frequencies corresponding to different tapered chains with the same N (between 15 and 20) but different q values (between 4 and 9 %) and finally compute different values of $k(\eta)$ for different η's. By averaging the values of k over the number of our trials and plotting the obtained average values of k versus the number of peaks η we finally arrived at the following linear expression:

$$k(\eta) = m\eta + b \ , \tag{18}$$

with the integer $\eta > 1$, the slope $m = 2.5705$ and the intercept $b = 1$. Systematic computational studies reveal that the resonance-like process in 1D chains with one of the edge grains subjected to a sinusoidally varying force, could be attained for a driving frequency of the following form:

$$\Omega = \frac{F_0^{1/6}}{R_{\text{large}}^{4/3}} \frac{1}{k(\eta, w)} \frac{1}{\rho^{1/2} D^{1/3}} \frac{1-(1-q)^{1/3}}{1-(1-q)^{N/3}} \ . \tag{19}$$

In order to test the accuracy of our results we have used Eq. (19) for different chain geometries (N, R_{large}, q) and different force magnitudes and found that the relative percentage error for Ω is less than 3% for small systems ($15 \le N \le 50$) and less than 6 % for large systems ($100 < N < 500$). We suspect that this disagreement could mostly be due to the way the restitution coefficient was defined in our computational studies and different approaches are under investigation.

Equation (19) shows that for a chosen configuration of a granular chain we can induce the resonance-like phenomenon by only modifying the amplitude of the driving force with the corresponding frequency. In principle, Eq (16) gives us a different kind of freedom, one in which the resonance-like frequency can be changed by adding or removing one or more of the grains from the system *Thus these discrete systems, which are intrinsically nonlinear and hence highly sensitive to external perturbations could be tuned to respond in a great many ways. This enormous parameter space across which the resonance-like process can be realized suggests* *that these systems could in principle serve as versatile sensors.* Our preliminary analyses suggest that frequencies between 0.1 Hz and 10^4 Hz can potentially be realized. Perhaps an even broader range is possible. To make the transition from these challenging mechanical systems to perhaps commercially available sensors will not be easy. We comment below on how the work presented here could potentially be useful in the context of energy harvesting applications. These comments are based upon our work in progress, which will be detailed in future journal publications.

5. Why is breathing of interest in energy harvesting applications?

Imagine an array of cylinders, each containing a system that is capable of breathing, and the array to be buried just under the surface of a beach such that one face of this array is exposed to the ocean waves as they break ashore. It may be possible to arrange this system to undergo sustainable breathing. If there are piezoelectric sensors panels at the surface of the cylinder array that is opposite to the ones exposed to the waves, then the mechanical energy of the waves can not only be efficiently absorbed, but transported into voltage drops arising due to the response of the piezoelectric sensor panel. Using a circuit such as a half-bridge power harvester with a leakage resistance, it may be possible to feed the voltage drop onto a power grid [24]. Thus, the energy of the ocean waves crashing ashore across large distances of the world's beaches could be harvestable using few moving parts. If we note that the power in the waves is estimated to be 1-10TW and that this is in the same ballpark as the world's power needs, harvesting the energy of dissipating waves could be a desirable source of energy [24]. However, one may be able to do better – because the complicated mechanical systems may actually be replaceable by fully electronic systems [24]. The following closing section elaborates on this point.

Nonlinear dynamics has begun to take shape as a research field in relatively recent times [4]. While a great many simple systems are nonlinear, isolating the effects of nonlinearities to understand and exploit its consequences is another matter. Hence, there is a strong need to have available experimental systems with a control on dissipation where the effects of nonlinearities can be systematically examined. Newcomb and coworkers have pioneered the development of circuits with low power requirements that can behave as nonlinear systems [28]. While the development of such circuits is challenging and time consuming, the potential benefits can be many, such as the development of chips that can effectively behave as complex nonlinear systems. Thus, given some simple input, these chips could be expected to output what a nonlinear system might yield as a solution.

We are currently studying the breathing problem with the focus to realize the lowest and highest frequencies and the range in between using Eq. (19). Our objective is to develop circuits that will be able to behave effectively as these nonlinear breathing systems. If successful, we expect these circuits to find many applications in energy harvesting technologies such as from wind, ocean waves, geothermal sources and more.

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Figure Captions

Fig. 1 The figure shows the potential (PE), kinetic (KE) and the total energy (TE) of a driven Ti6Al4V monodispersed chain versus time. Here N= 15, R_{large} = 5mm and F_0 = 100 N. The inset shows the PE behavior over a longer time span

Fig. 2 The figure shows the behaviour of $TF^{1/6}$ versus q for systems of two different sizes (N= 15, 20). The results are obtained simulationally for F_0 values of 100 N, 200 N, 400 N, and 800 N. The plot basically shows how the chain length shrinks with increasing q and this behavior agrees with the form shown in section 4.1

Fig. 3 Space versus time plots of kinetic energy of grains (shown in grey scale with darker implying more kinetic energy) for monodispersed (q = 0%) chain of N=15 Ti6Al4V beads with R = 5 mm. Here F_0 = 100 N, w= 0.01 for a range of different driving frequency (**Ω**) values

Fig. 4 Total, kinetic and potential energy versus time behaviour in monodispersed chains of N=15 Ti6Al4V beads at driving frequencies **Ω**= 2570 rads/sec which is below (upper panel) and at **Ω**= 3420 rads/s which is above (lower panel) the non-linear resonance frequency of **Ω**= 2990 rads/s

Fig. 5 Non-linear resonant behaviour of monodispersed tungsten bead chain with N= 15, R = 5mm, w= 0.01, F = 100N and **Ω**= 2171 rads/s and η=1

Fig. 6 Two distinct non-linear resonant behavior patterns corresponding to η = 2, 3 (upper, lower panels) for beads of Ti6Al4V with q = 5%, N= 15, F = 100 N, R = 5mm, and w= 0.01. For η=2, **Ω**= 1709 rads/s (upper panel) and for η=3, **Ω**= 1200 rads/s (lower panel)

Table 1 Definitions of mathematical symbols used

Symbol	Definition
c_{average}	Average energy transport speed in the chain
D	$\frac{3}{2}\frac{(1-\sigma^2)}{Y}$
F_0	Amplitude of applied force on the first grain
F_{loading}	Force on the grains during loading
$F_{\text{unloading}}$	Force on the grains during unloading
q	Tapering coefficient
R_i	Radius of the i^{th} grain
T	Round-trip energy transport time in the chain
w	Restitution coefficient
Y	Young's modulus
z_i	Position of the grain center in the unperturbed state of the chain
$\delta_{i,i+1}$	Overlap between adjacent grains i and $i+1$
η	Number of peaks in each breathing cycle
ρ	Density of elastic grain
σ	Poisson ratio
Ω	Driving and non-linear breathing frequency at non-linear resonance

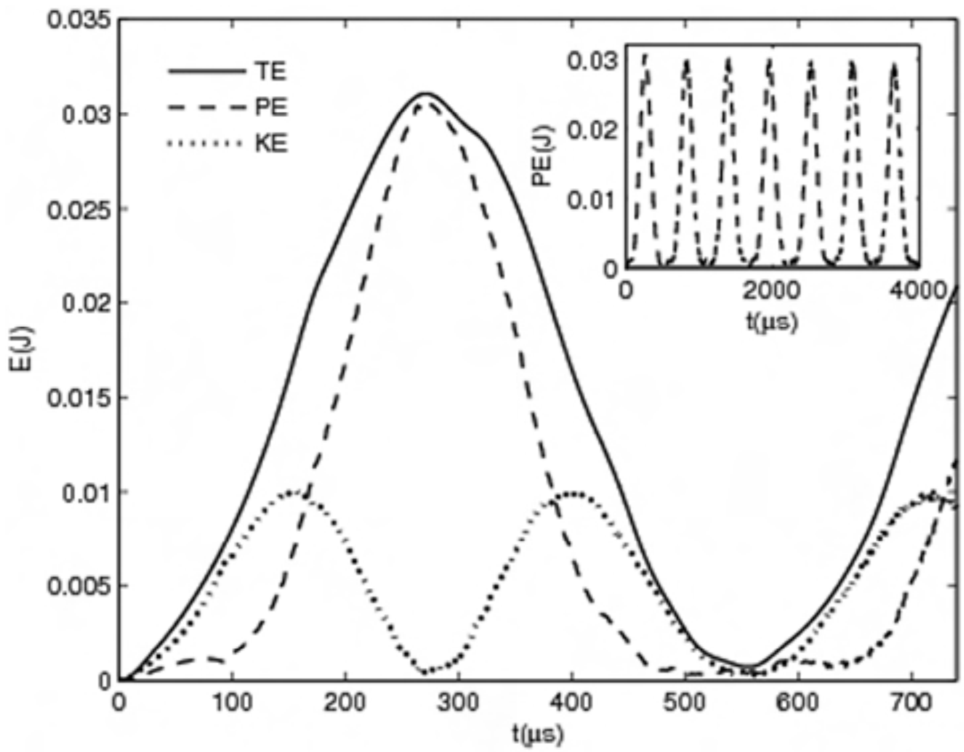
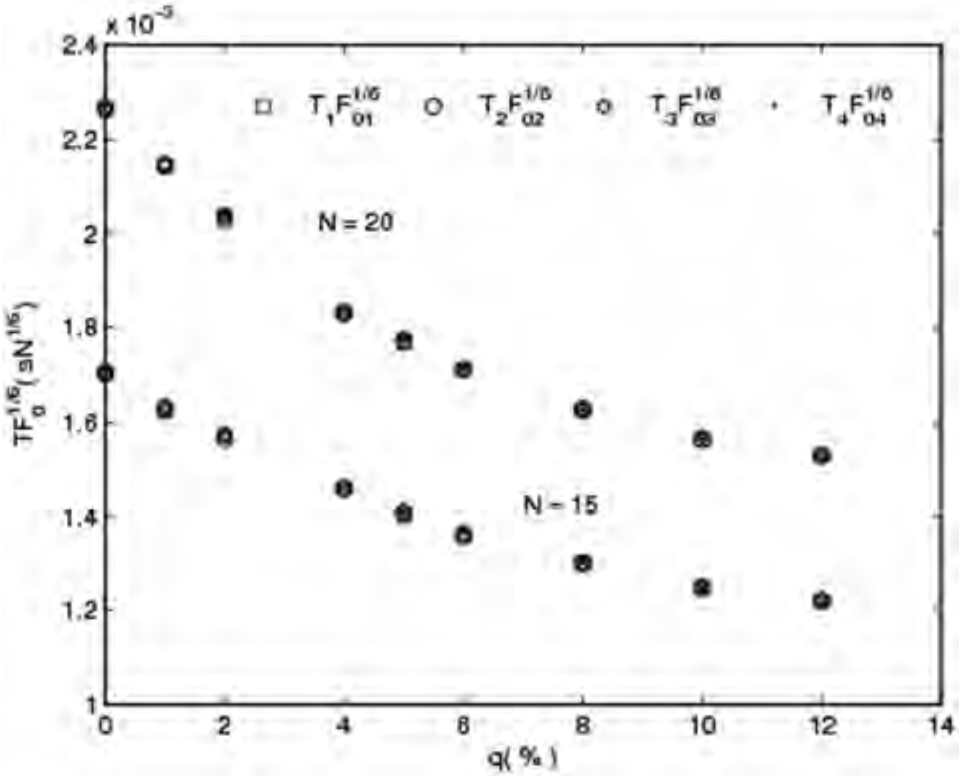
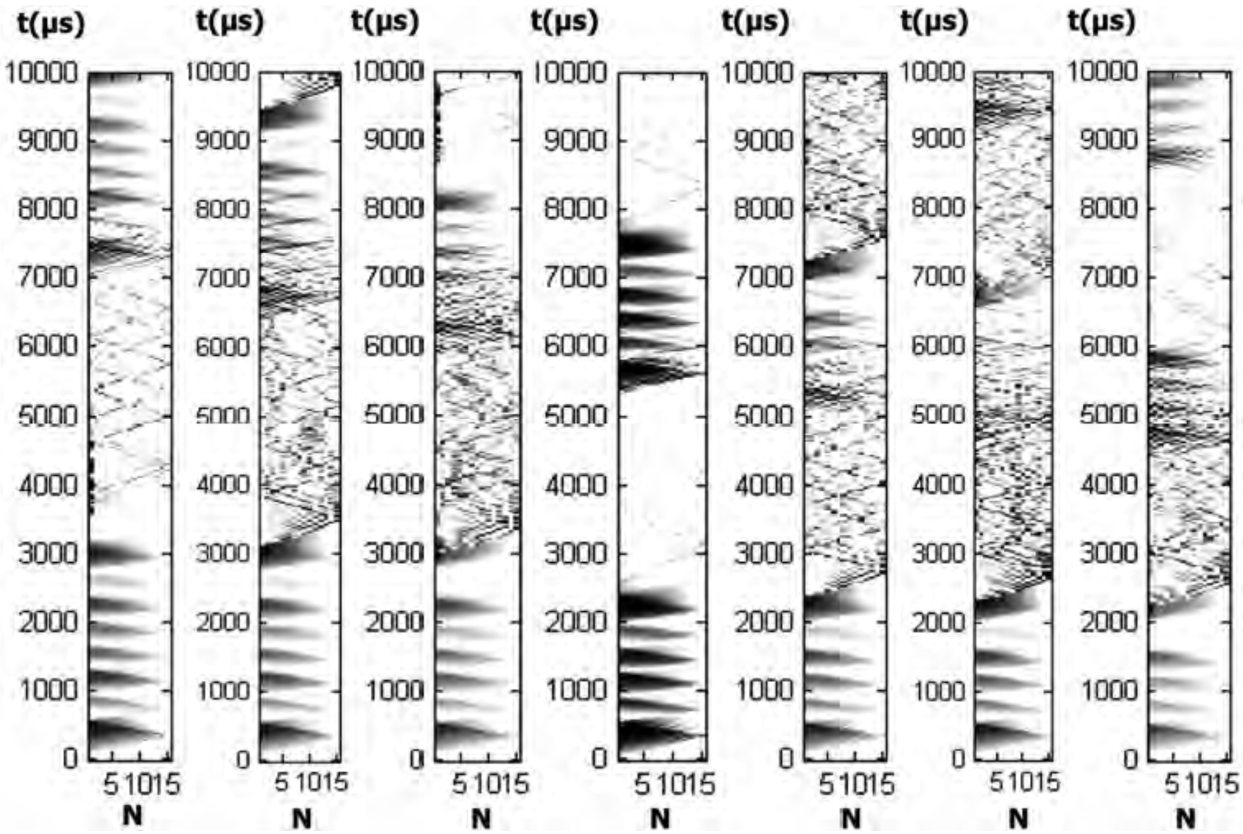

0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
E(J)
TE
PE
KE
0
100
200
300
400
500
600
700
t(μs)
0.03
0.02
0.01
0
PE(J)
0
2000
4000
t(μs)

Fig. 1



x 10^-3
2.4
2.2
2
1.8
1.6
1.4
1.2
1
$T_1F_{01}^{1/6}$ $T_2F_{02}^{1/6}$ $T_3F_{03}^{1/6}$ $T_4F_{04}^{1/6}$
N = 20
N = 15
$TF_0^{1/6}(\approx N^{1/6})$
0
2
4
6
8
10
12
14
q(%)

Fig. 2



t(μs)
t(μs)
t(μs)
t(μs)
t(μs)
t(μs)
t(μs)
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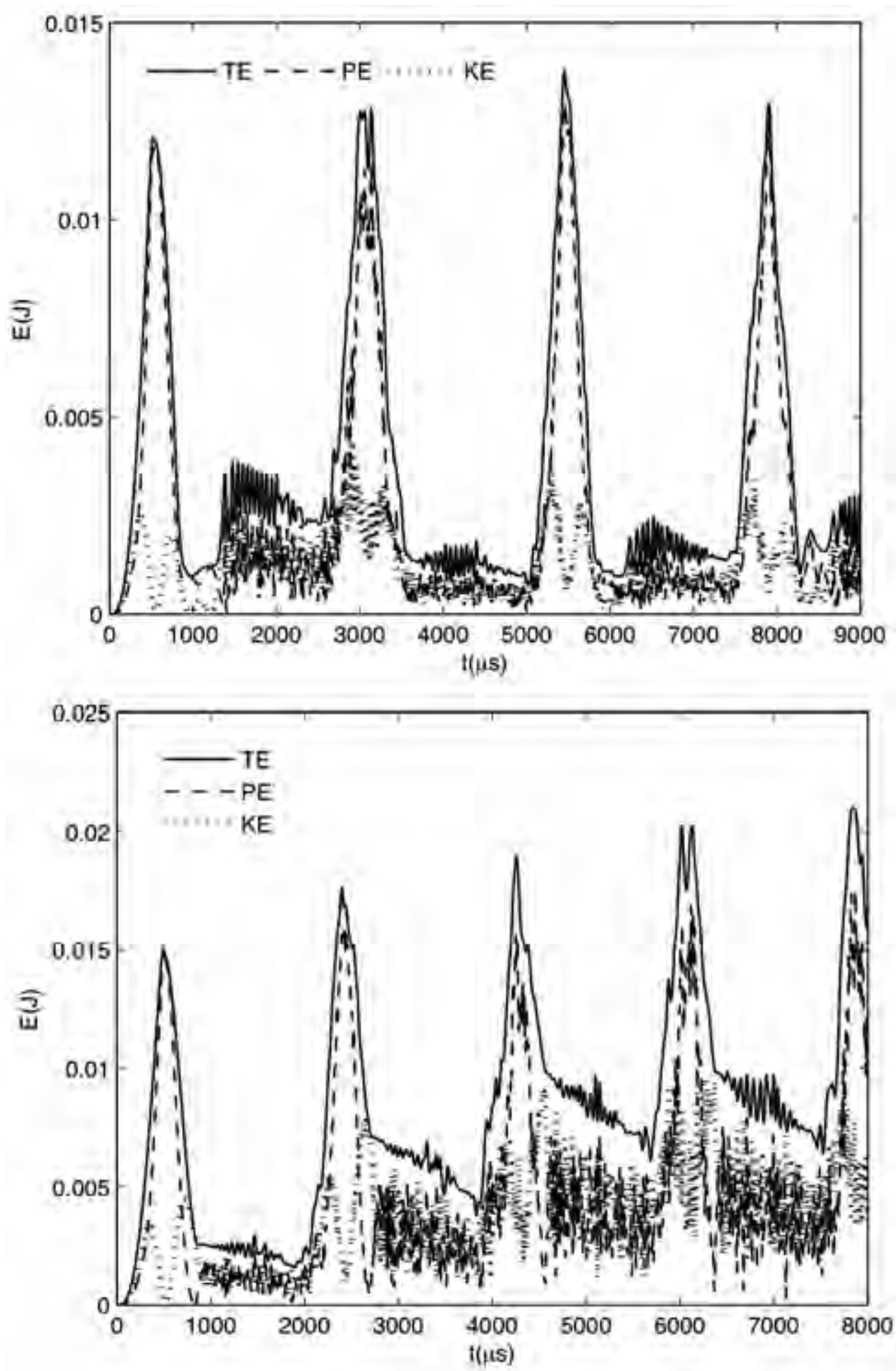
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
5 10 15
N

Fig. 3



TE
PE
KE
E(J)
t(μs)
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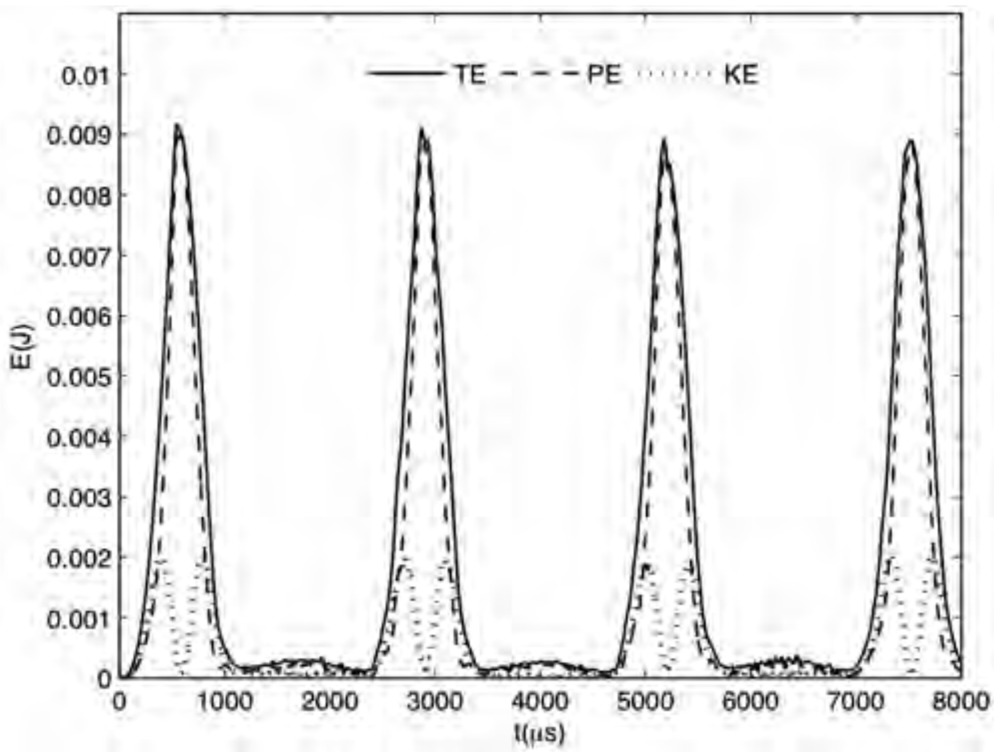
0.01
0.005
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
0.025
0.02

Fig. 4



TE
PE
KE
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0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
E(J)
0
1000
2000
3000
4000
5000
6000
7000
8000
t(μs)

Fig. 5

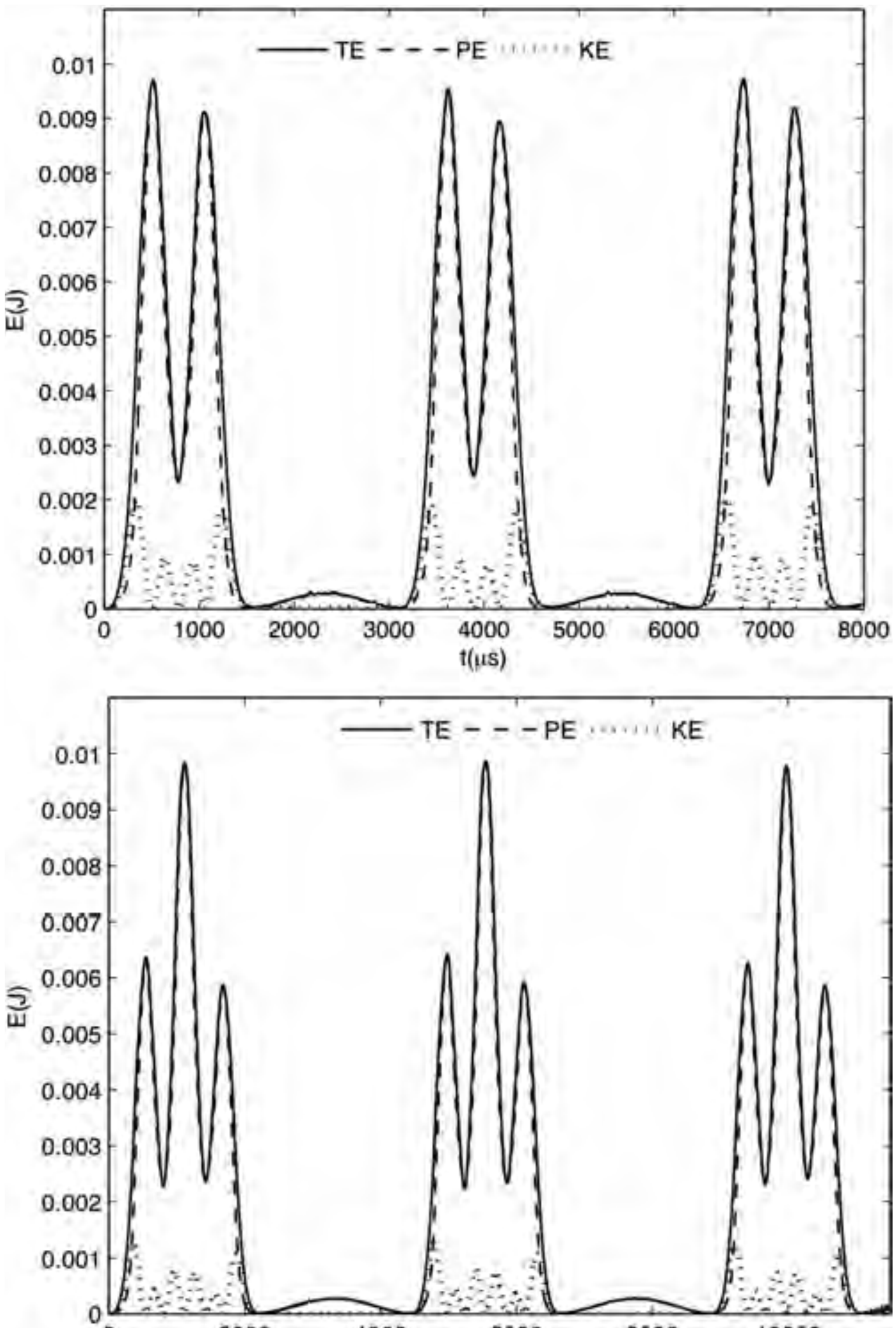

Fig 6